परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

नोट

(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **27** हैं ।

(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।

(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।

(IV) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें ।**

(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

NOTE

(I) Please check that this question paper contains **27** printed pages.

(II) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(III) Please check that this question paper contains **38** questions.

(IV) **Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**

(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

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गणित (बुनियादी)
MATHEMATICS (BASIC)

निर्धारित समय : *3 घण्टे*

Time allowed : 3 hours

अधिकतम अंक : *80*

Maximum Marks : 80

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) इस प्रश्न-पत्र में **38** प्रश्न हैं। **सभी** प्रश्न **अनिवार्य** हैं।

(ii) यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ**।

(iii) **खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **1** अंक के प्रश्न हैं।

(iv) **खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **2** अंकों के प्रश्न हैं।

(v) **खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **3** अंकों के प्रश्न हैं।

(vi) **खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **5** अंकों के प्रश्न हैं।

(vii) **खण्ड ङ** में प्रश्न संख्या **36** से **38** तक प्रकरण अध्ययन आधारित **4** अंकों के प्रश्न हैं। प्रत्येक प्रकरण अध्ययन में आंतरिक विकल्प **2** अंकों के प्रश्न में दिया गया है।

(viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 2 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 3 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।

(ix) जहाँ आवश्यक हो स्वच्छ आकृतियाँ बनाइए। जहाँ आवश्यक हो $\pi = \dfrac{22}{7}$ लीजिए, यदि अन्यथा न दिया गया हो।

(x) कैल्कुलेटर का उपयोग **वर्जित** है।

खण्ड क

इस खण्ड में **20** बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न **1** अंक का है। $20 \times 1 = 20$

1. एक वृत्त, जिसकी परिधि 22 cm है, के एक चतुर्थांश का परिमाप है :

(A) 29 cm

(B) 22 cm

(C) 12·5 cm

(D) 5·5 cm

2. एक शंकु और एक बेलन एक ही ऊँचाई और एक ही त्रिज्या के हैं। शंकु के आयतन का अनुपात बेलन के आयतन से है :

(A) 1 : 1   (B) 1 : 3

(C) 3 : 1   (D) 1 : 2

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i)   *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii)   *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii)   *In **Section A**, Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv)   *In **Section B**, Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v)   *In **Section C**, Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi)   *In **Section D**, Questions no. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

(vii)   *In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each. Internal choice is provided in **2** marks questions in each case study.*

(viii)   *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.*

(ix)   *Draw neat diagrams wherever required. Take $\pi = \dfrac{22}{7}$ wherever required, if not stated.*

(x)   *Use of calculator is **not** allowed.*

SECTION A

*This section has **20** Multiple Choice Questions (MCQs) carrying **1** mark each. 20×1=20*

1.   The perimeter of a quadrant of a circle of circumference 22 cm is :

     (A)   29 cm

     (B)   22 cm

     (C)   12·5 cm

     (D)   5·5 cm

2.   A cone and cylinder have same height and same radius. The volume of the cone and the volume of the cylinder are in the ratio :

     (A)   1 : 1   (B)   1 : 3

     (C)   3 : 1   (D)   1 : 2

3. निम्नलिखित सारणी में एक कक्षा के 23 विद्यार्थियों द्वारा प्राप्त अंकों को दर्शाया गया है।

प्राप्तांक	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
विद्यार्थियों की संख्या	5	3	4	8	3

बहुलक वर्ग की निचली सीमा है :

(A) 10 (B) 20

(C) 30 (D) 40

4. किसी बंटन के लिए, यदि माध्य = 15 और बहुलक = 12 है, तो उसका माध्यक होगा :

(A) 12

(B) 13

(C) 14

(D) 15

5. पासों का एक युग्म एक साथ उछाला जाता है। माना E घटना ''दोनों पासों पर आई संख्याओं का योगफल कम-से-कम 9 है'' को निरूपित करती है। घटना E के संभव परिणामों की संख्या है :

(A) 4

(B) 6

(C) 10

(D) 26

6. यदि $p = 2^3 \times 3^2 \times 5$ और $q = 2^2 \times 3^3$ है, तो p और q का LCM है :

(A) $2^3 \times 3^3$

(B) $2^2 \times 3^2$

(C) $2^2 \times 3^2 \times 5$

(D) $2^3 \times 3^3 \times 5$

7. 3^n, जहाँ n एक प्राकृत संख्या है, जिस अंक पर कभी-भी समाप्त नहीं होता है, वह अंक है :

(A) 3 (B) 5

(C) 7 (D) 9

3. The following table shows the marks scored by 23 students of a class.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of Students	5	3	4	8	3

The lower limit of the modal class is :

(A)   10   (B)   20

(C)   30   (D)   40

4. For a distribution, if mean = 15 and mode = 12, then its median is :

(A)   12

(B)   13

(C)   14

(D)   15

5. A pair of dice is thrown simultaneously. Let E denote the event that "The sum of numbers obtained on both dice is at least 9." The number of outcomes in favour of event E is :

(A)   4

(B)   6

(C)   10

(D)   26

6. If p = $2^3 \times 3^2 \times 5$ and q = $2^2 \times 3^3$, then the LCM of p and q is :

(A) $2^3 \times 3^3$

(B) $2^2 \times 3^2$

(C) $2^2 \times 3^2 \times 5$

(D) $2^3 \times 3^3 \times 5$

7. 3^n, where n is a natural number, cannot end with the digit :

(A)   3   (B)   5

(C)   7   (D)   9

• • •

8. एक अभाज्य संख्या के :

   (A) केवल दो ही अभाज्य गुणनखंड होते हैं

   (B) केवल एक ही अभाज्य गुणनखंड होता है

   (C) कम-से-कम एक अभाज्य गुणनखंड होता है

   (D) कम-से-कम दो अभाज्य गुणनखंड होते हैं

9. दिया गया रैखिक समीकरण निकाय :

   $$x = a \quad \text{और} \quad y = b$$

   (A) एक अद्वितीय हल के साथ संगत है।

   (B) अपरिमित रूप से अनेक हल के साथ संगत है।

   (C) दो हलों के साथ संगत है।

   (D) असंगत है।

10. यदि $(x + 1)^3 = x^3 + 1$ को एक द्विघात समीकरण $px^2 + qx + r = 0$ के रूप में व्यक्त किया जाए, तो $p - q + r$ का मान होगा :

   (A) 0   (B) 1

   (C) 3   (D) 6

11. यदि बिन्दुओं (3, 5) तथा (−1, −1) को जोड़ने वाले रेखाखंड का मध्य-बिन्दु (a, 2b) है, तो (a, b) बराबर है :

   (A) (1, 2)   (B) (2, 2)

   (C) (2, 1)   (D) (1, 1)

12. y-अक्ष से बिंदु (− 3, 4) की दूरी है :

   (A) − 3   (B) 3

   (C) 4   (D) 5

13. θ का मान जिसके लिए $\sin 2\theta = \tan 45°$ है :

   (A) 22·5°   (B) 30°

   (C) 45°   (D) 90°

8. A prime number has :

    (A)   exactly two prime factors

    (B)   exactly one prime factor

    (C)   at least one prime factor

    (D)   at least two prime factors

9. The system of linear equations given by x = a and y = b is :

    (A)   Consistent with a unique solution.

    (B)   Consistent with infinitely many solutions.

    (C)   Consistent with two solutions.

    (D)   Inconsistent.

10. If $(x + 1)^3 = x^3 + 1$ is expressed as a quadratic equation in the form $px^2 + qx + r = 0$, then the value of $p - q + r$ is :

    (A)   0   (B)   1

    (C)   3   (D)   6

11. If point (a, 2b) is the mid-point of the line segment joining the points (3, 5) and (−1, −1), then (a, b) is equal to :

    (A)   (1, 2)   (B)   (2, 2)

    (C)   (2, 1)   (D)   (1, 1)

12. The distance of point (− 3, 4) from y-axis is :

    (A)   − 3   (B)   3

    (C)   4   (D)   5

13. The value of θ for which $\sin 2\theta = \tan 45°$ is :

    (A)   22·5°   (B)   30°

    (C)   45°   (D)   90°

• • •

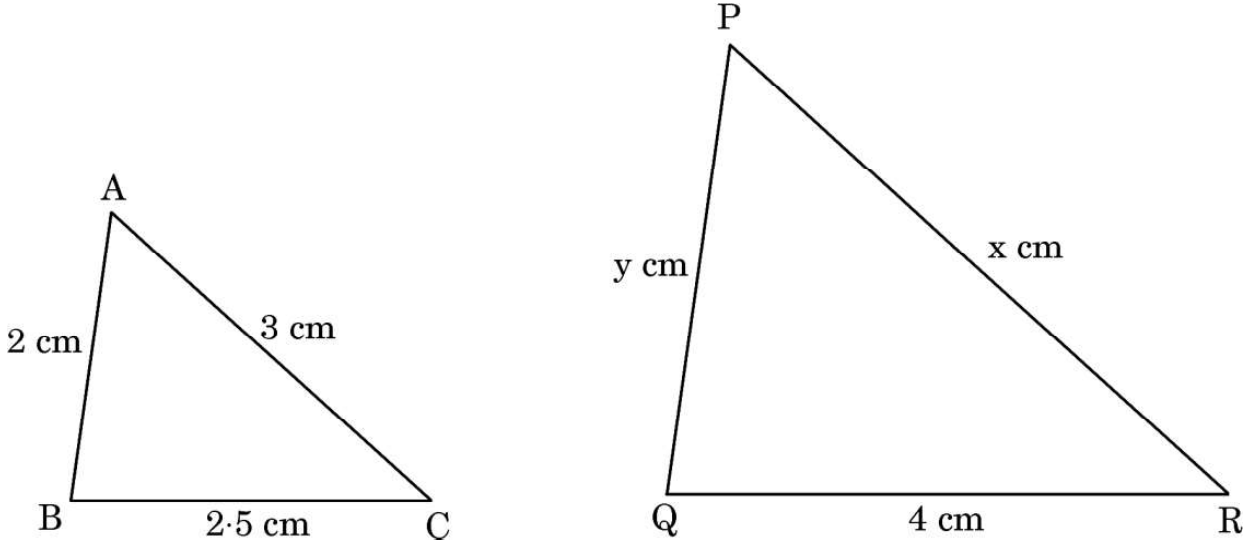**14.** निम्नलिखित में से किस प्रकार के त्रिभुज सदैव समरूप होते हैं ?
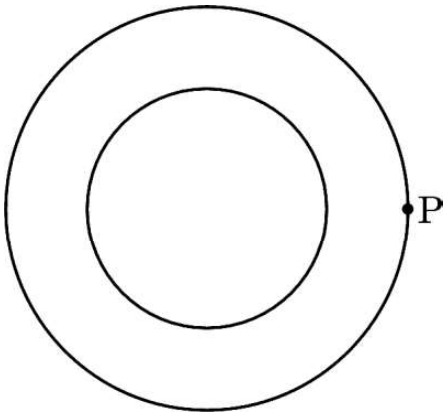
(A) समकोण त्रिभुज

(B) न्यूनकोण त्रिभुज

(C) समद्विबाहु त्रिभुज

(D) समबाहु त्रिभुज

15. नीचे दी गई आकृतियों में, x और y के किन मानों के लिए $\triangle ABC$ और $\triangle QRP$ समरूप होंगे ?

(A) $x = 6,\ y = 5$

(B) $x = 5,\ y = 6$

(C) $x = 6,\ y = 6$

(D) $x = 12,\ y = 3.2$

16. दी गई आकृति में, बाह्य वृत्त पर स्थित बिंदु P से अन्तः वृत्त पर कितनी स्पर्श-रेखाएँ खींची जा सकती हैं ?

(A) 0   (B) 1

(C) 2   (D) अनंत

14. Which types of triangles are always similar ?
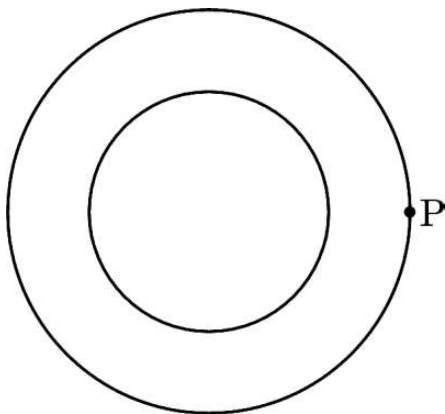
(A) Right-angled triangles

(B) Acute-angled triangles

(C) Isosceles triangles

(D) Equilateral triangles

15. What values of x and y will make Δ ABC similar to Δ QRP in the figures given below ?
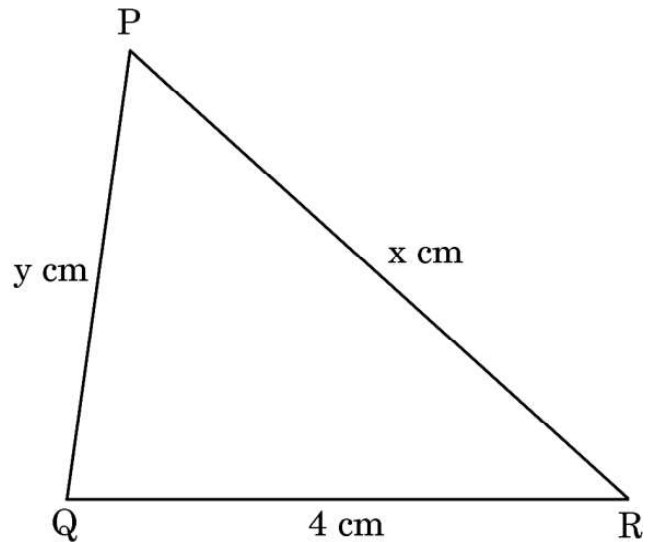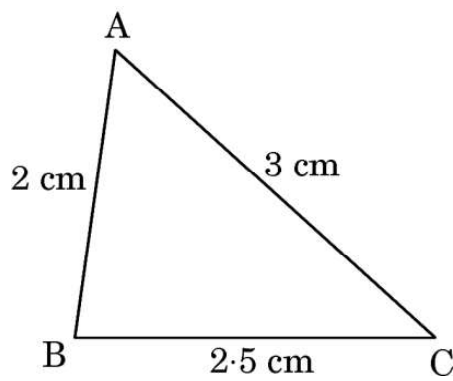
(A) x = 6, y = 5

(B) x = 5, y = 6

(C) x = 6, y = 6

(D) x = 12, y = 3·2

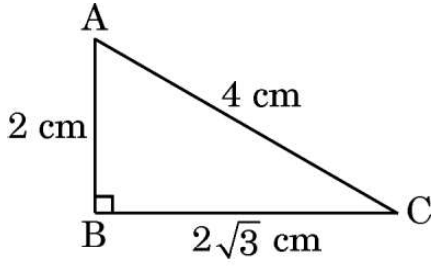**16.** How many tangents can be drawn from the point P on the outer circle to the inner circle in the given figure ?

(A) 0   (B) 1

(C) 2   (D) Infinite

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17. दी गई आकृति में, बिंदु C से बिंदु A का उन्नयन कोण क्या है ?

(A)   30°   (B)   45°

(C)   60°   (D)   ज्ञात नहीं किया जा सकता है

18. किसी वृत्त के एक त्रिज्यखंड का कोण जिसका क्षेत्रफल, वृत्त के क्षेत्रफल का एक-आठवाँ भाग है, है :

(A) $22\dfrac{1}{2}^\circ$ (B)   45°

(C)   60°   (D)   90°

प्रश्न संख्या 19 और 20 अभिकथन एवं तर्क आधारित प्रश्न हैं । दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है । इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए ।

(A)   अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है ।

(B)   अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या **नहीं** करता है ।

(C)   अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है ।

(D)   अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है ।

19.   *अभिकथन (A) :*   एक कोण θ के लिए, sec θ = 1 $\Rightarrow$ tan θ = 0.

    तर्क (R) : $\sec^2 \theta + \tan^2 \theta = 1.$
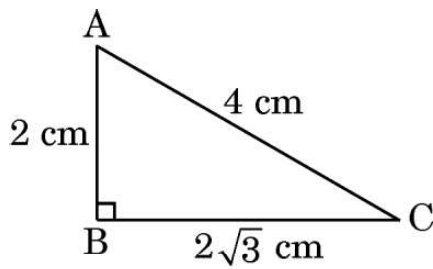
20.   *अभिकथन (A) :*   प्रत्येक द्विघात समीकरण के दो वास्तविक मूल होते हैं ।

    तर्क (R) :   एक द्विघात बहुपद के अधिक-से-अधिक दो शून्यक हो सकते हैं ।

17. In the given figure, the angle of elevation of point A from point C is :

    (A)   30°   (B)   45°

    (C)   60°   (D)   Cannot be determined

18. The angle of the sector of a circle whose area is one-eighth of the area of the circle is :

    (A) $22\dfrac{1}{2}^{\circ}$ (B)   45°

    (C)   60°   (D)   90°

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.*

    (A)   Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

    (B)   Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of Assertion (A).

    (C)   Assertion (A) is true, but Reason (R) is false.

    (D)   Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* For an angle θ, sec θ = 1 ⇒ tan θ = 0.

    Reason (R) : $\sec^2 \theta + \tan^2 \theta = 1$.

20. *Assertion (A) :* Every quadratic equation has two real roots.

    Reason (R) :   A quadratic polynomial can have at most two zeroes.

खण्ड ख

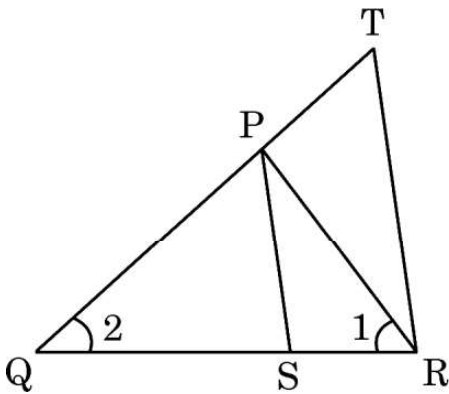*इस खण्ड में 5 अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।*   *5×2 =10*

21. 10 cm त्रिज्या की एक वृत्ताकार चादर से एक चतुर्थांश काटा जाता है। बची हुई चादर का परिमाप ज्ञात कीजिए।

22. निम्नलिखित रैखिक समीकरण निकाय का हल ग्राफ़ द्वारा ज्ञात कीजिए :

$x + y = 5$ और $x - y = 3$

23. (क) 10 m ऊँचे एक ऊर्ध्वाधर खम्भे की धरती पर पड़ने वाली छाया की लम्बाई 15 m है और उसी समय पर, एक मीनार धरती पर 45 m की छाया बनाता है। मीनार की ऊँचाई ज्ञात कीजिए।

अथवा

(ख) दी गई आकृति में, $\dfrac{QR}{QS} = \dfrac{QT}{PR}$ और $\angle 1 = \angle 2$ है। सिद्ध कीजिए कि

$\Delta PQS \sim \Delta TQR$

SECTION B

*This section has **5** Very Short Answer (VSA) type questions carrying **2** marks each.*   *5×2=10*

21. From a circular sheet of radius 10 cm, a quadrant is cut. Find the perimeter of the remaining sheet.

22. Solve the following system of linear equations graphically :

$x + y = 5$ and $x - y = 3$

23. (a) A vertical pole of height 10 m casts a shadow of 15 m on the ground and at the same time, a tower casts a shadow of 45 m on the ground. Find the height of the tower.
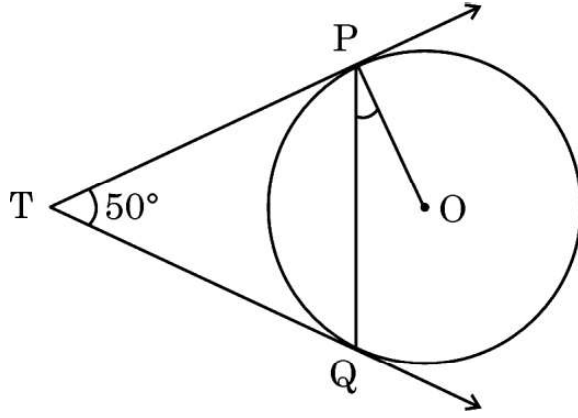
OR

(b) In the given figure, $\dfrac{QR}{QS} = \dfrac{QT}{PR}$ and $\angle 1 = \angle 2$. Prove that $\Delta PQS \sim \Delta TQR$.

• • •
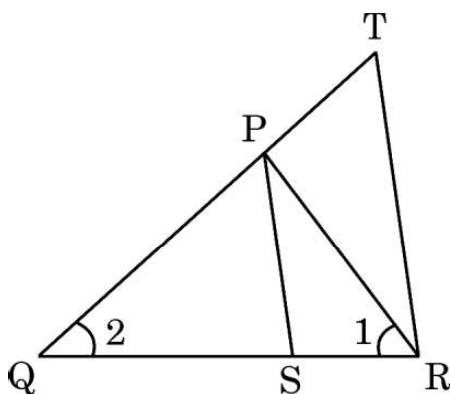
24. दी गई आकृति में, TP और TQ दो स्पर्श-रेखाएँ हैं। यदि $\angle$ PTQ = 50° है, तो $\angle$ OPQ की माप ज्ञात कीजिए।

25. (क) यदि $\sin 3A = 1$ है, तो $\cos 2A - \tan^2 45°$ का मान ज्ञात कीजिए।

अथवा

(ख) यदि $(\sec A + \tan A)(1 - \sin A) = k \cos A$ है, तो k का मान ज्ञात कीजिए।

खण्ड ग

*इस खण्ड में **6** लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के **3** अंक हैं।*   *6×3 =18*
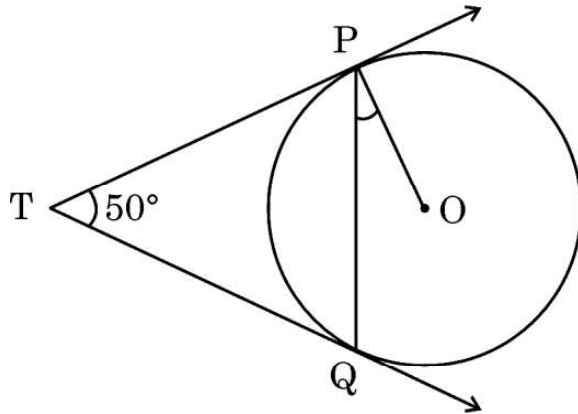
26. एक डिब्बे में 6 नीले, 4 सफेद और 8 लाल कंचे हैं। एक कंचा इस डिब्बे से यादृच्छया निकाला जाता है। प्रायिकता ज्ञात कीजिए कि निकाला गया कंचा :

(i) सफेद है

(ii) सफेद या लाल है

(iii) लाल नहीं है

24. In the given figure, TP and TQ are two tangents. If ∠ PTQ = 50°, then find the measure of ∠ OPQ.

25. (a) If $\sin 3A = 1$, then find the value of $\cos 2A - \tan^2 45°$.

OR

(b) If $(\sec A + \tan A)(1 - \sin A) = k \cos A$, then find the value of k.

SECTION C

*This section has **6** Short Answer (SA) type questions carrying **3** marks each.*   *6×3=18*

26. A box contains 6 blue, 4 white and 8 red marbles. A marble is drawn at random from this box. Find the probability that the marble so drawn is :

(i) white

(ii) white or red

(iii) not red

27. दी गई आकृति में, एक चतुर्भुज ABCD में एक वृत्त बना है जो चतुर्भुज की भुजाओं AB, BC, CD और DA को क्रमश: बिन्दुओं P, Q, R तथा S पर स्पर्श करता है । सिद्ध कीजिए कि $\angle$ AOB + $\angle$ COD = 180°.
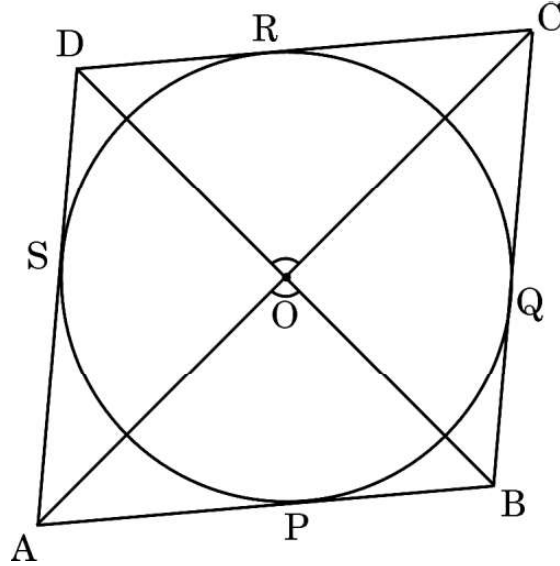
28. (क) सिद्ध कीजिए कि $\sqrt{2}$ एक अपरिमेय संख्या है ।

अथवा

(ख) ज्ञात कीजिए कि निम्नलिखित संख्याओं a, b और c में से कौन-सी भाज्य संख्या/संख्याएँ है/हैं ।

$$a = 7 \times 11 \times 13 + 13$$
$$b = 6 \times 5 \times 4 + 4$$
$$c = 7 \times 13 + 6$$

29. बहुपद $25a^2 - 10a + 1$ के शून्यक ज्ञात कीजिए और दिए गए बहुपद के शून्यकों और गुणांकों के बीच संबंध की सत्यता की जाँच कीजिए ।

30. (क) एक भिन्न $\frac{1}{3}$ हो जाता है, जब उसके अंश से 1 घटाया जाता है और वह $\frac{1}{4}$ हो जाता है, जब उसके हर में 8 जोड़ दिया जाता है । वह भिन्न ज्ञात कीजिए ।

अथवा

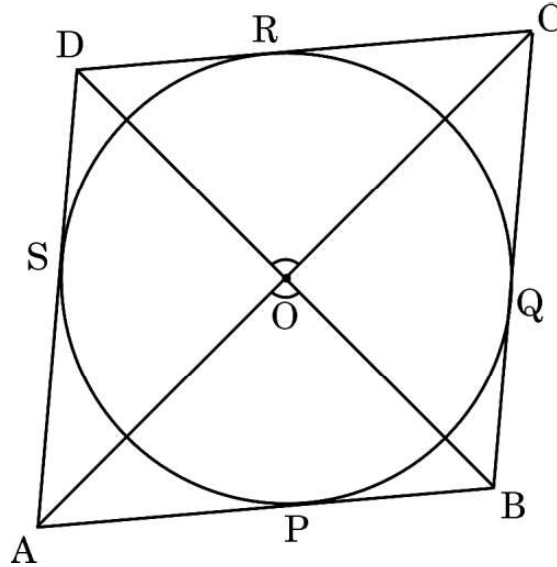(ख) k का वह मान ज्ञात कीजिए जिससे निम्नलिखित रैखिक समीकरणों के युग्म के अपरिमित रूप से अनेक हल हैं :

$$kx + 3y - (k - 3) = 0 \quad \text{और} \quad 12x + ky - k = 0$$

इस प्रकार प्राप्त समीकरण युग्म के कोई दो हल ज्ञात कीजिए ।

· · ·

27. In the given figure, a circle is inscribed in a quadrilateral ABCD which touches the sides AB, BC, CD and DA at P, Q, R and S respectively. Prove that ∠ AOB + ∠ COD = 180°.

28. (a) Prove that $\sqrt{2}$ is an irrational number.

OR

(b) Find which among the following numbers a, b and c is/are composite numbers.

$$a = 7 \times 11 \times 13 + 13$$
$$b = 6 \times 5 \times 4 + 4$$
$$c = 7 \times 13 + 6$$

29. Find the zeroes of the polynomial $25a^2 - 10a + 1$ and verify the relationship between the zeroes and coefficients of the given polynomial.

30. (a) A fraction becomes $\frac{1}{3}$, when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$, when 8 is added to its denominator. Find the fraction.

OR

(b) Find the value of k for which the following pair of linear equations will have infinitely many solutions :

$$kx + 3y - (k - 3) = 0 \text{ and } 12x + ky - k = 0$$

Hence, find any two solutions of the given pair of equations.

• • •

31. निम्नलिखित त्रिकोणमितीय सर्वसमिका को सिद्ध कीजिए :

$$\frac{\tan\theta}{1+\cot\theta} + \frac{\cot\theta}{1+\tan\theta} = \tan\theta + \cot\theta - 1$$

खण्ड घ

इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं। $4\times5=20$

32. निम्नलिखित तालिका में एक मोहल्ले के 25 परिवारों के दैनिक खर्चे का विवरण दिया गया है।

दैनिक खर्च (₹ में)	परिवारों की संख्या
500 – 750	4
750 – 1000	2x + 1
1000 – 1250	12
1250 – 1500	x
1500 – 1750	2

x का मान ज्ञात कीजिए और इसके पश्चात माध्य दैनिक खर्च भी ज्ञात कीजिए।

33. **(क)** दो क्रमागत विषम पूर्णांक ज्ञात कीजिए जिनके वर्गों का योगफल 290 है।

अथवा

(ख) एक दानी (चैरीटी) ट्रस्ट फैसला करता है कि उन्हें एक आयताकार हॉल, जिसका क्षेत्रफल 300 m² हो, बनाना है। हॉल की लम्बाई, इसकी चौड़ाई के दुगने से एक मीटर अधिक होनी है। हॉल की लम्बाई और चौड़ाई ज्ञात कीजिए।

31. Prove the following trigonometric identity :

$$\frac{\tan\theta}{1+\cot\theta} + \frac{\cot\theta}{1+\tan\theta} = \tan\theta + \cot\theta - 1$$

SECTION D

*This section has **4** Long Answer (LA) type questions carrying **5** marks each.   4×5=20*

32. The following table shows the daily expenditure of 25 households of a locality.

Daily Expenditure (in ₹)	Number of Households
500 – 750	4
750 – 1000	2x + 1
1000 – 1250	12
1250 – 1500	x
1500 – 1750	2

Find the value of x. Hence find the mean daily expenditure.

33. (a)   Find two consecutive odd integers, sum of whose squares is 290.

OR

(b)   A charity trust decides to build a rectangular hall having an area of 300 m². The length of the hall is one metre more than twice its width. Find the length and breadth of the hall.
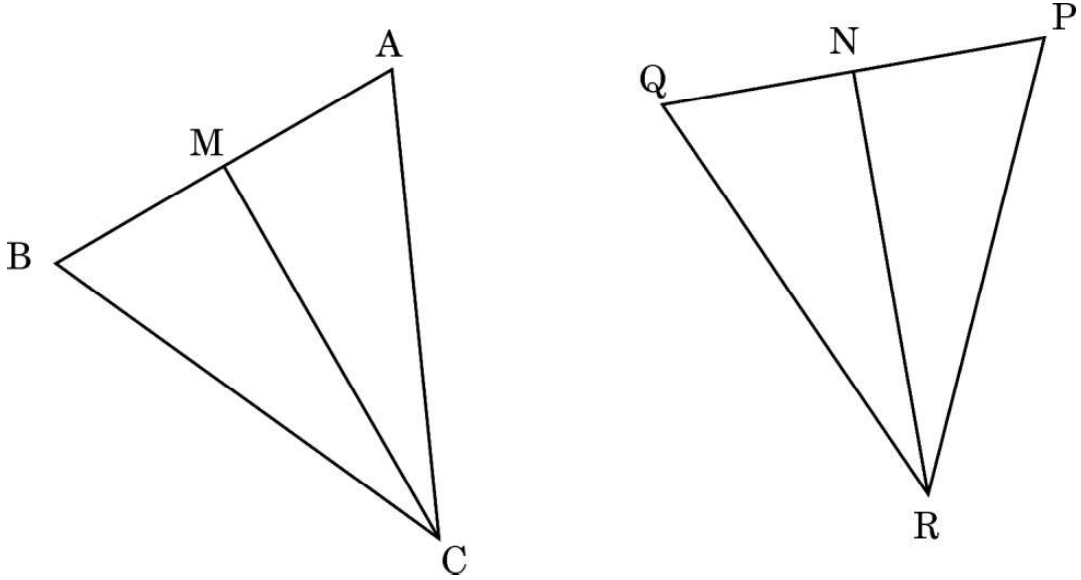
34. (क) ''आधारभूत समानुपातिकता प्रमेय'' लिखिए और उसे सिद्ध भी कीजिए।

अथवा

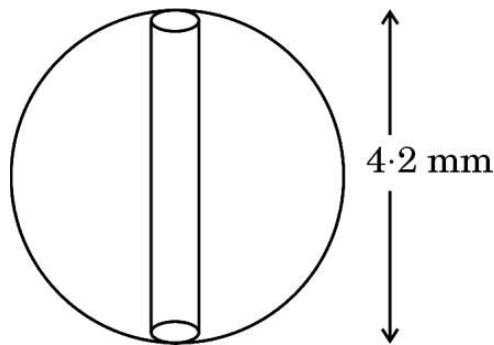(ख) दी गई आकृति में, CM और RN क्रमश: त्रिभुजों ABC और PQR की माध्यिकाएँ हैं। यदि ∆ ABC ~ ∆ PQR है, तो सिद्ध कीजिए कि :

(i)   ∆ AMC ~ ∆ PNR

(ii)   ∠ BCM = ∠ QRN

(iii)   ∆ BMC ~ ∆ QNR

35. एक हार लकड़ी के बीड्स से बना है। प्रत्येक बीड 4·2 mm व्यास के गोले के रूप में है। प्रत्येक बीड से एक बेलन खोद कर निकाला गया है। यदि बेलन की त्रिज्या 1 mm है, तो प्रत्येक बीड में बची लकड़ी का आयतन ज्ञात कीजिए।
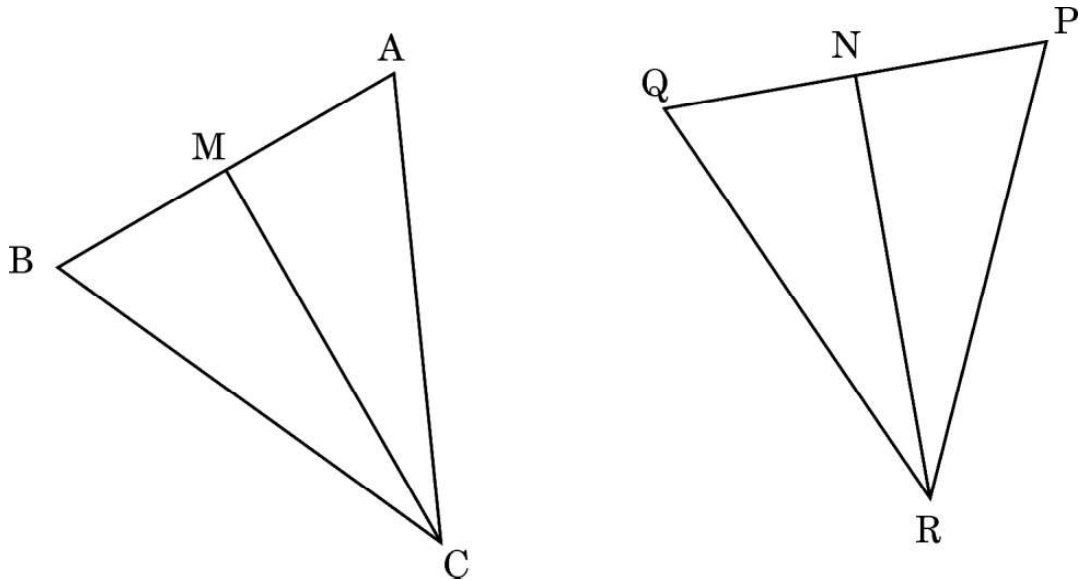

4·2 mm

34. (a) State and Prove "Basic Proportionality Theorem".

<p align="center">OR</p>
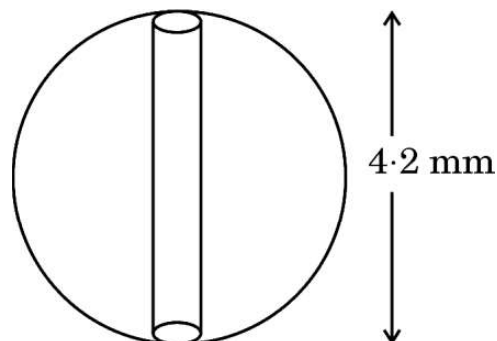
(b) In the given figure, CM and RN are respectively, the medians of $\triangle$ ABC and $\triangle$ PQR. If $\triangle$ ABC ~ $\triangle$ PQR, prove that :

(i) $\triangle$ AMC ~ $\triangle$ PNR

(ii) $\angle$ BCM = $\angle$ QRN

(iii) $\triangle$ BMC ~ $\triangle$ QNR

35. A necklace is made up of wooden beads. Each bead is in the form of a sphere of diameter 4·2 mm. A cylinder is hollowed out from each bead. If the radius of the cylinder is 1 mm, find the volume of wood left in each bead.
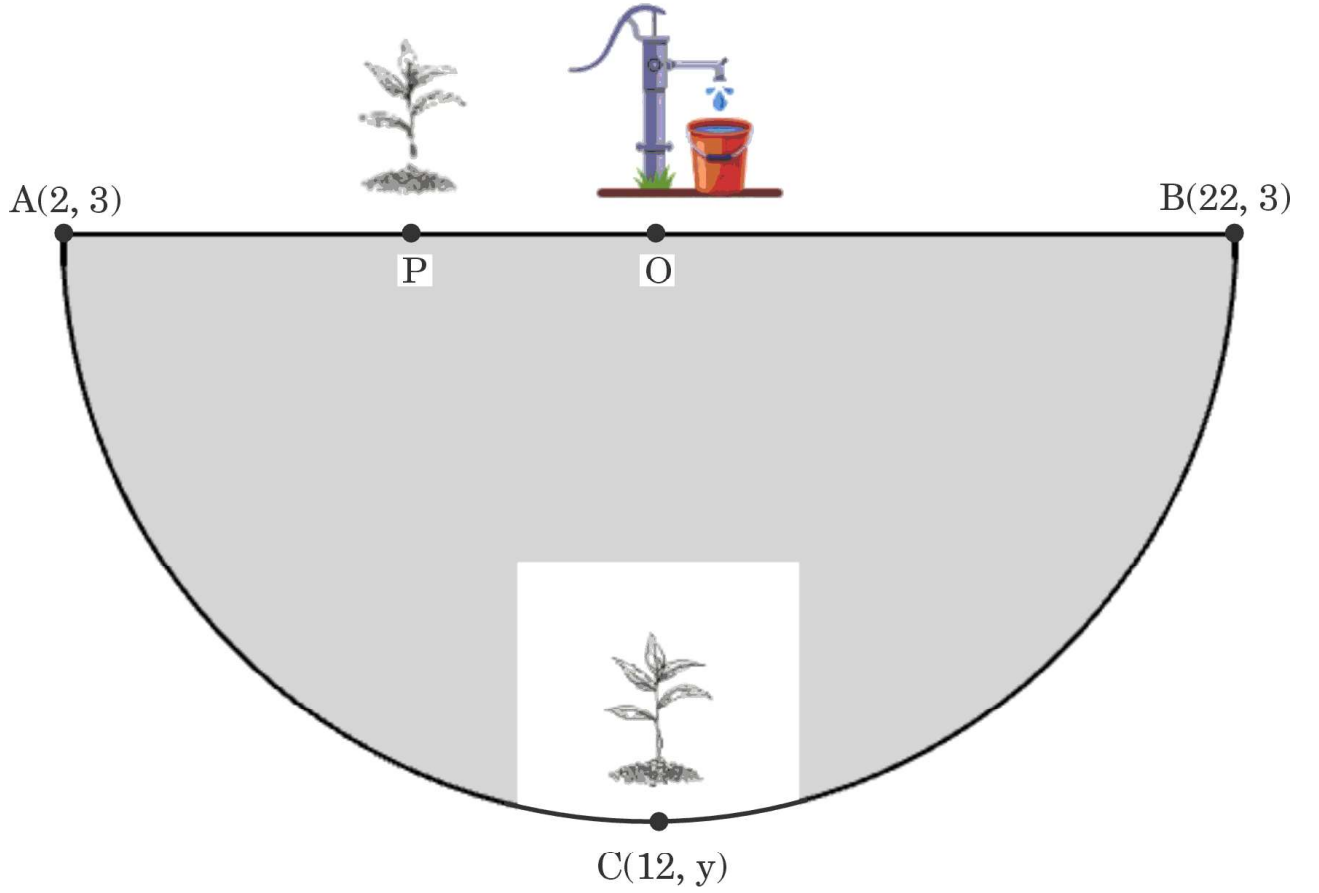

4·2 mm

खण्ड ङ

इस खण्ड में **3** प्रकरण अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के **4** अंक हैं।   *3×4=12*

प्रकरण अध्ययन – 1

36. अमन की सोसाइटी में एक अर्धवृत्ताकार पार्क है। वह इस पार्क की परिधि (परिसीमा) पर पौधे लगाना चाहता है। जैसा कि नीचे चित्र में दिखाया गया है, पार्क के केन्द्र O पर व्यास AB के अनुदिश एक बोरवेल है।

A(2, 3)   B(22, 3)

P   O

C(12, y)

उपर्युक्त जानकारी के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i)		बिन्दु O के निर्देशांक ज्ञात कीजिए।	*1*
(ii)		अर्धवृत्ताकार पार्क की त्रिज्या ज्ञात कीजिए।	*1*
(iii)	(क)	एक पौधा बिन्दु C(12, y) पर लगाया गया है। C के निर्देशांक ज्ञात कीजिए।	*2*

अथवा

(ख)   व्यास AB के अनुदिश, एक पौधा बिन्दु P पर इस प्रकार लगाया गया है कि

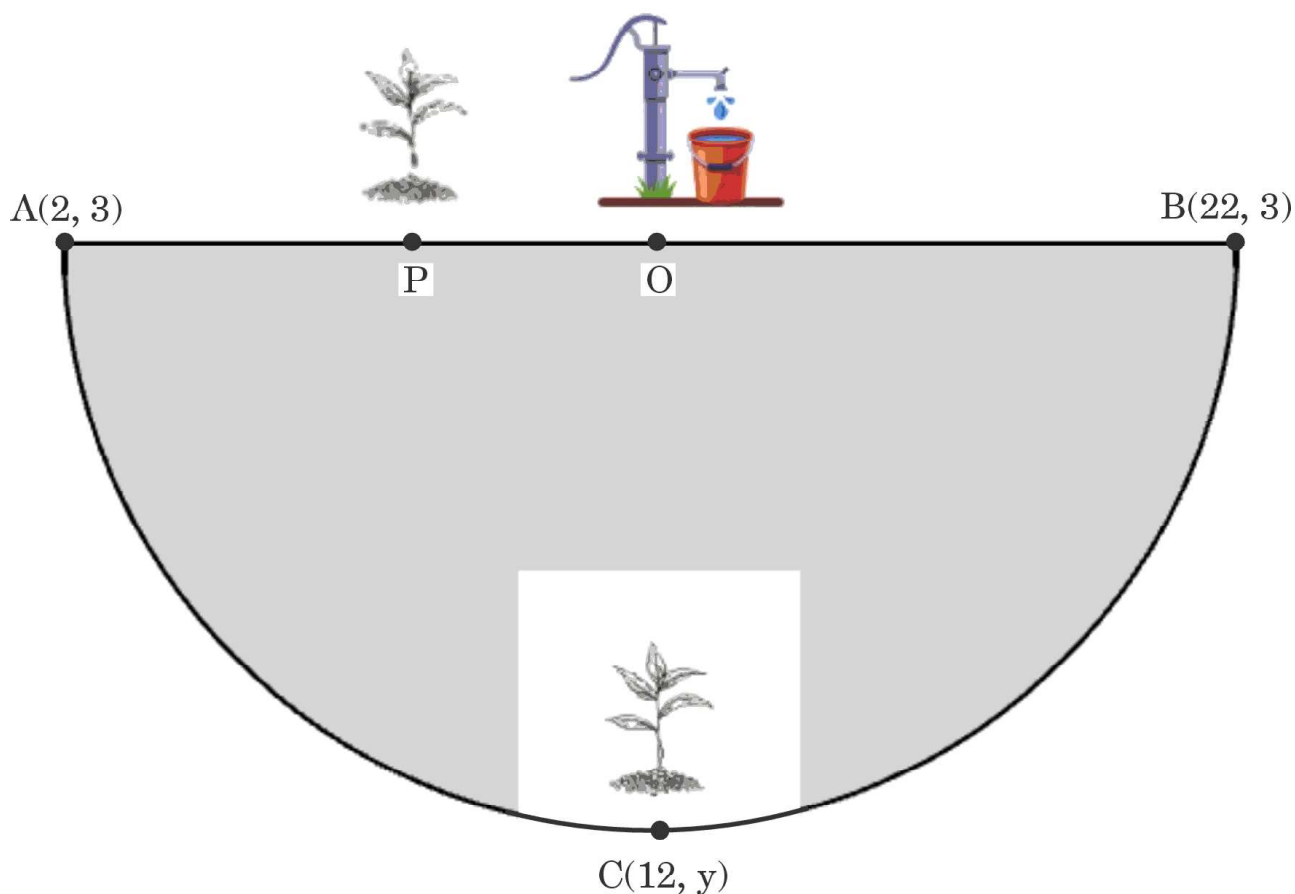PA = $\frac{1}{3}$ PB. बिंदु P के निर्देशांक ज्ञात कीजिए।   *2*

SECTION E

*This section has **3** case study based questions carrying **4** marks each.*   *3×4=12*

Case Study – 1

36. There is a semicircular park in Aman's society. He wishes to plant saplings along the boundary of the park. There is a borewell at the centre O of the park along the diameter AB as shown in the figure below.

A(2, 3)   B(22, 3)

P   O

C(12, y)

Based on the above information, answer the following questions :

(i)   Find the coordinates of point O.   1

(ii)   Find the radius of the semicircular park.   1

(iii)   (a)   One sapling is kept at point C(12, y). Find the coordinates of C.   2

OR

(b)   One sapling is kept at point P along AB so that PA = $\frac{1}{3}$ PB.

Find the coordinates of P.   2

प्रकरण अध्ययन – 2

37. एक सोसाइटी में लोगों को स्वस्थ जीवन शैली जीने के लिए प्रशिक्षित करने के लिए एक योग प्रशिक्षक को नियुक्त किया गया था। सोसाइटी पार्क में प्रतिदिन योग सत्र शाम 5 बजे से 7 बजे तक आयोजित किए जाते थे। पहले दिन, 5 व्यक्ति योग सत्र में शामिल हुए, दूसरे दिन 3 और व्यक्ति योग सत्र में शामिल हुए, तीसरे दिन 3 और व्यक्ति योग सत्र में शामिल हुए और इसी तरह हर दिन 3 और व्यक्ति शामिल होते चले गए।

उपर्युक्त जानकारी के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) किस दिन, 59 व्यक्तियों ने योग सत्र में हिस्सा लिया ? *1*

(ii) 31वें दिन कितने व्यक्तियों ने योग सत्र में हिस्सा लिया ? *1*

(iii) (क) योग प्रशिक्षक को प्रति सत्र, प्रति व्यक्ति ₹ 100 दिए गए। किस दिन प्रशिक्षक को ₹ 5,000 प्राप्त होंगे ? *2*

अथवा

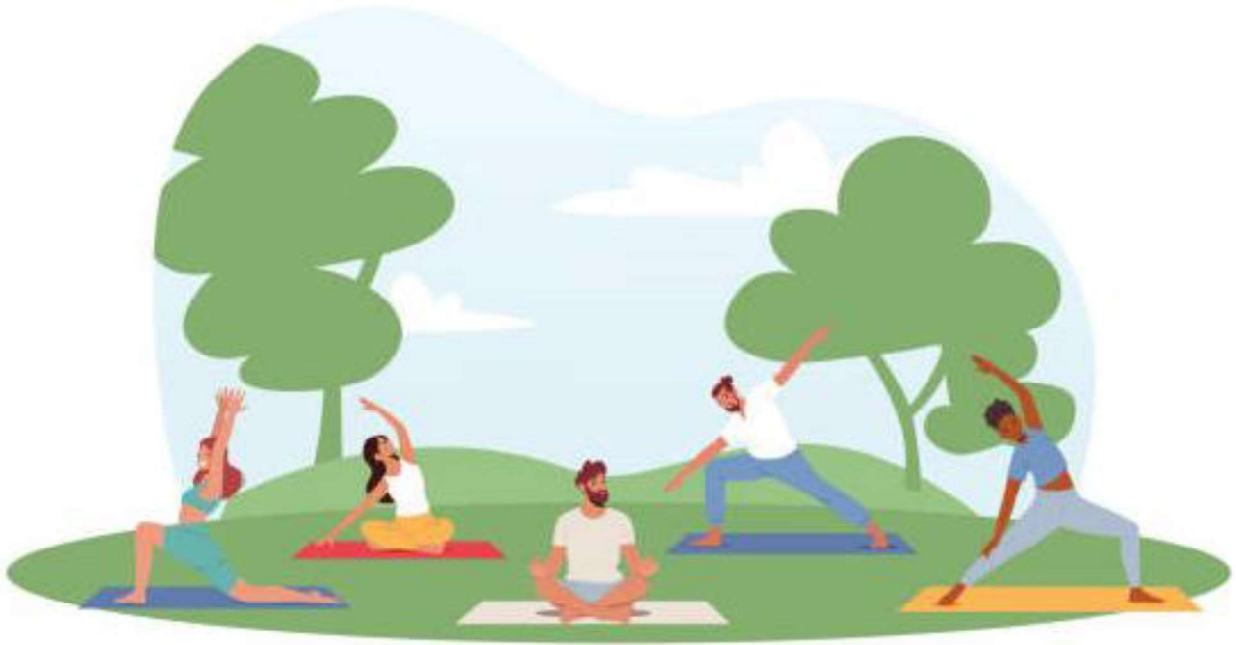(ख) योग प्रशिक्षक ने पहले 16 दिनों में कुल कितने रुपए प्राप्त किए ? *2*

Case Study – 2

37. In a society, a yoga instructor was hired to train the people of the society to live a healthy lifestyle. Yoga sessions were held daily from 5 p.m. to 7 p.m. in the society park. On day one, 5 people joined the yoga session, on day two, 3 more people joined, on day three, another 3 people joined and in this manner every next day, 3 more people kept on joining.

Based on the given information, answer the following questions :

(i)   On which day did 59 people join the yoga session ?   *1*

(ii) How many people joined the yoga session on the 31st day ?   *1*

(iii) (a) The yoga instructor was paid ₹ 100 for each person attending the yoga session. On which day would he earn ₹ 5,000 ?   *2*
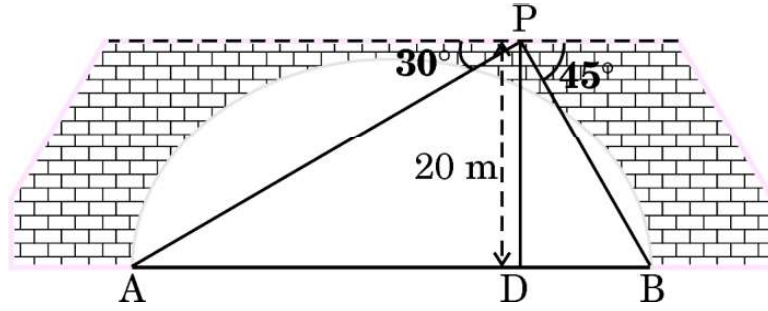
OR

(b) What was the total amount earned by the yoga instructor in 16 days ?   *2*

प्रकरण अध्ययन – 3

38. दो मोटर नावें A और B नदी के विपरीत किनारों पर दूसरी ओर पहुँचने के लिए प्रतीक्षा कर रही हैं । नदी से 20 m ऊपर पुल पर एक बिन्दु P से, नावों के अवनमन कोण क्रमशः 30° और 45° हैं, जैसा कि चित्र में नीचे दिखाया गया है । दोनों नावें समान समय पर क्रमशः 10 m/s और 5 m/s की गति से रवाना होती हैं ।

उपर्युक्त जानकारी के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) बिन्दु P के ठीक नीचे नदी में बिन्दु D तक पहुँचने के लिए नाव A द्वारा तय की गई दूरी ज्ञात कीजिए । ($\sqrt{3}$ = 1·73 प्रयोग कीजिए) *1*

(ii) नदी की चौड़ाई कितनी है ? *1*

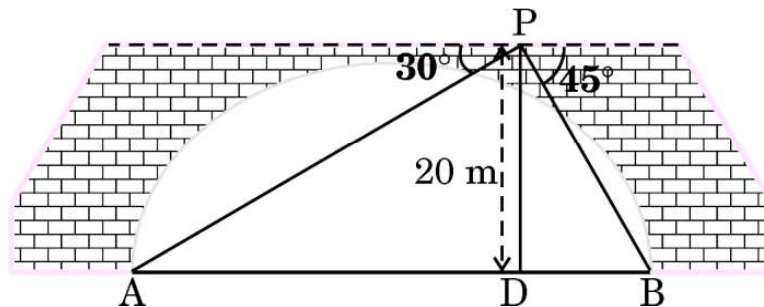(iii) (क) कौन-सी नाव बिन्दु D पर पहले पहुँचेगी और वह दूसरी नाव से कितने समय पहले पहुँचेगी ? *2*

अथवा

(ख) 3 सेकण्ड पश्चात दोनों नावों के बीच की दूरी कितनी है ? *2*

Case Study – 3

38. Two motorboats A and B are waiting at the opposite banks of a river in order to reach the opposite side. From a point P on the bridge, 20 m above the river, the angles of depression of the boats are 30° and 45° respectively, as shown in the figure given below. Both the boats leave at the same time at the speed of 10 m/s and 5 m/s, respectively

Based on the above information, answer the following questions :

(i) Find the distance travelled by boat A to reach point D in the river, vertically below the point P. (Use $\sqrt{3}$ =1·73) *1*

(ii) What is the width of the river ? *1*

(iii) (a) Which boat will reach point D first, and how much earlier, than the other boat ? *2*

OR

(b) What is the distance between the two boats after 3 seconds ? *2*

	Marking Scheme **Strictly Confidential** **(For Internal and Restricted use only)** **Secondary School Examination, 2025** **SUBJECT NAME MATHEMATICS (BASIC) (Q.P. CODE 430/3/3)**

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, evaluation done and several other aspects. It's leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in** **News Paper/Website etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-X, while evaluating two competency-based** **questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking scheme carries only suggested value points for the answers. These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.
10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer.

	● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.	
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.	
15	Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.	
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.	
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.	
18	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.	

Set 430/3/3

MARKING SCHEME
MATHEMATICS (BASIC)

SECTION A

*This section has **20** Multiple Choice Questions (MCQs) carrying **1** mark each.*   *20×1=20*

1. The perimeter of a quadrant of a circle of circumference 22 cm is :

    (A)   29 cm

    (B)   22 cm

    (C)   12·5 cm

    (D)   5·5 cm

Ans: (C) 12·5 cm	1

2. A cone and cylinder have same height and same radius. The volume of the cone and the volume of the cylinder are in the ratio :

    (A)   1 : 1   (B)   1 : 3

    (C)   3 : 1   (D)   1 : 2

Ans: (B) 1 : 3	1

3. The following table shows the marks scored by 23 students of a class.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of Students	5	3	4	8	3

The lower limit of the modal class is :

    (A)   10   (B)   20

    (C)   30   (D)   40

Ans: (C) 30	1
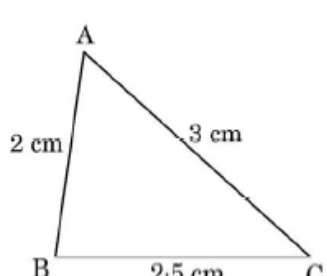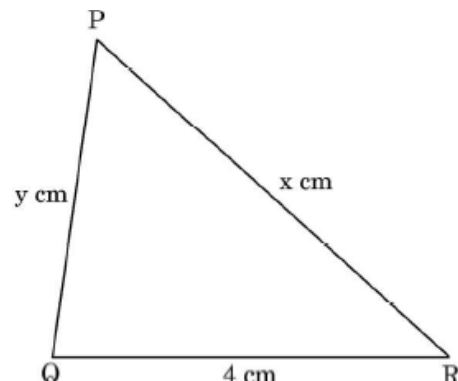
4. For a distribution, if mean = 15 and mode = 12, then its median is :

    (A)   12

    (B)   13

    (C)   14

    (D)   15

Ans: (C) 14	1

5. A pair of dice is thrown simultaneously. Let E denote the event that "The sum of numbers obtained on both dice is at least 9." The number of outcomes in favour of event E is :

(A)　4

(B)　6

(C)　10

(D)　26

| **Ans:** (C)　10 | 1 |

6. If $p = 2^3 \times 3^2 \times 5$ and $q = 2^2 \times 3^3$, then the LCM of p and q is :

(A)　$2^3 \times 3^3$

(B)　$2^2 \times 3^2$

(C)　$2^2 \times 3^2 \times 5$

(D)　$2^3 \times 3^3 \times 5$

| **Ans:** (D)　$2^3 \times 3^3 \times 5$ | 1 |

7. 3^n, where n is a natural number, cannot end with the digit :

(A)　3　　　　　　　　　　　(B)　5

(C)　7　　　　　　　　　　　(D)　9

| **Ans:** (B)　5 | 1 |

8. A prime number has :

(A)　exactly two prime factors

(B)　exactly one prime factor

(C)　at least one prime factor

(D)　at least two prime factors

| **Ans:** (B)　exactly one prime factor | 1 |

9. The system of linear equations given by x = a and y = b is :

(A)　Consistent with a unique solution.

(B)　Consistent with infinitely many solutions.

(C)　Consistent with two solutions.

(D)　Inconsistent.

| **Ans:** (A)　Consistent with a unique solution | 1 |

**10.** If $(x + 1)^3 = x^3 + 1$ is expressed as a quadratic equation in the form $px^2 + qx + r = 0$, then the value of $p - q + r$ is :

   (A)   0   (B)   1

   (C)   3   (D)   6

Ans: (A)   0   1

11. If point $(a, 2b)$ is the mid-point of the line segment joining the points $(3, 5)$ and $(-1, -1)$, then (a, b) is equal to :

   (A)   (1, 2)   (B)   (2, 2)

   (C)   (2, 1)   (D)   (1, 1)

Ans: (D)   (1, 1)   1

12. The distance of point $(-3, 4)$ from y-axis is :

   (A) -3 (B)   3

   (C)   4   (D)   5

Ans: (B)   3   1

13. The value of θ for which $\sin 2\theta = \tan 45°$ is :

   (A)   22·5°   (B)   30°

   (C)   45°   (D)   90°

Ans: (C)   45°   1

14. Which types of triangles are always similar ?
   (A)   Right-angled triangles
   (B)   Acute-angled triangles
   (C)   Isosceles triangles
   (D)   Equilateral triangles

Ans: (D)   Equilateral triangles   1

15. What values of x and y will make Δ ABC similar to Δ QRP in the figures given below ?

(A)   x = 6, y = 5
(B)   x = 5, y = 6
(C)   x = 6, y = 6
(D)   x = 12, y = 3·2

Ans: (B)   x = 5, y = 6   1

16.   How many tangents can be drawn from the point P on the outer circle to
      the inner circle in the given figure ?
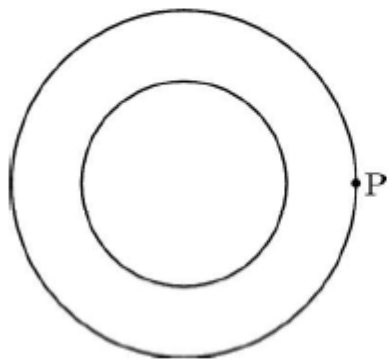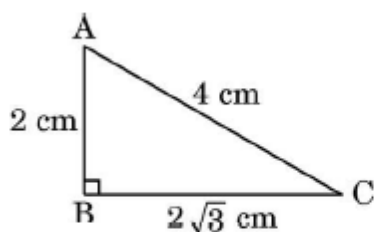

(A)   0   (B)   1
(C)   2   (D)   Infinite

Ans: (C)   2   1

17.   In the given figure, the angle of elevation of point A from point C is :

(A)   30°   (B)   45°
(C)   60°   (D)   Cannot be determined

Ans: (A)   30°   1

18.   The angle of the sector of a circle whose area is one-eighth of the area of
      the circle is :

(A) $22\frac{1}{2}^{\circ}$ (B)   45°

(C)   60°   (D)   90°

Ans: (B)   45°   1

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below.

   (A)   Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

   (B)   Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of Assertion (A).

   (C)   Assertion (A) is true, but Reason (R) is false.

   (D)   Assertion (A) is false, but Reason (R) is true.

19.   *Assertion (A)* : For an angle θ, $\sec \theta = 1 \Rightarrow \tan \theta = 0$.

   Reason (R) : $\sec^2 \theta + \tan^2 \theta = 1$.

Ans: (C) Assertion (A) is true, but Reason (R) is false	1

20.   *Assertion (A)* : Every quadratic equation has two real roots.

   Reason (R) :   A quadratic polynomial can have at most two zeroes.

Ans: (D) Assertion (A) is false, but Reason (R) is True.	1

SECTION B

This section has 5 Very Short Answer (VSA) type questions carrying 2 marks each.   5×2=10

21.   From a circular sheet of radius 10 cm, a quadrant is cut. Find the perimeter of the remaining sheet.

Solution: Perimeter of the remaining sheet

$$= 2\pi r - \frac{1}{4} \times 2\pi r + 2r \ = \ \frac{3}{2}\pi r + 2r$$

$$= \frac{3}{2} \times \frac{22}{7} \times 10 + 20 \qquad 1$$

$$= \frac{470}{7} \text{ cm} \quad \text{or} \quad 67.14 \text{ cm} \qquad 1$$
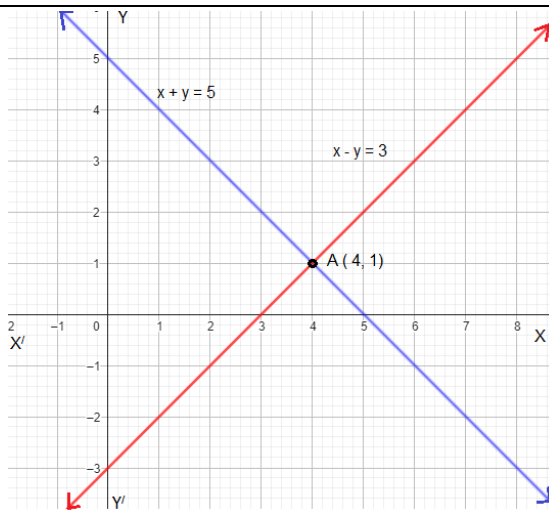
22.   Solve the following system of linear equations graphically :

   $x + y = 5$ and $x - y = 3$

Solution:

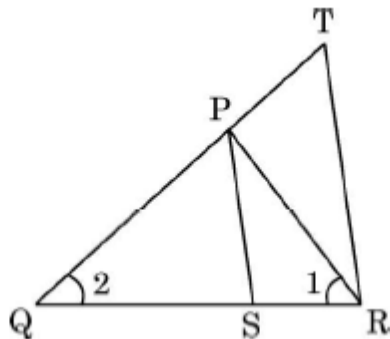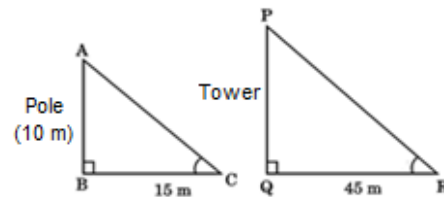   Correct graph of equations   1½

23. **(a)** A vertical pole of height 10 m casts a shadow of 15 m on the ground and at the same time, a tower casts a shadow of 45 m on the ground. Find the height of the tower.

OR

(b) In the given figure, $\dfrac{QR}{QS} = \dfrac{QT}{PR}$ and $\angle 1 = \angle 2$. Prove that

$\triangle PQS \sim \triangle TQR$.

Solution: (a)

$\because \triangle ABC \sim \triangle PQR$ ½

$\therefore \dfrac{10}{PQ} = \dfrac{15}{45} \Rightarrow PQ = 30$ 1½

Height of the tower = 30 m

OR
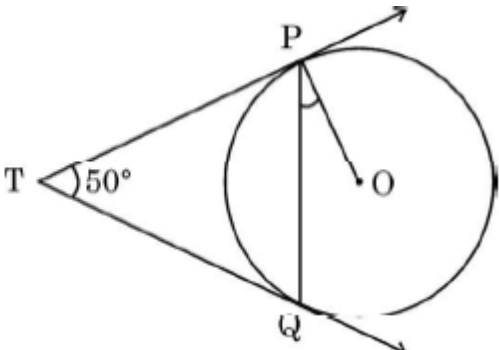
(b) In $\triangle PQR \angle 1 = \angle 2 \Rightarrow PQ = PR$ ½

∴ In ΔPQS and ΔTQR	
∴ $\dfrac{QR}{QS} = \dfrac{QT}{PR} \Rightarrow \dfrac{QR}{QS} = \dfrac{QT}{QP}$	½
also ∠Q = ∠Q (Common)	½
∴ Δ PQS ~ Δ TQR (by SAS similarity criterion)	½

24. In the given figure, TP and TQ are two tangents. If ∠ PTQ = 50°, then find the measure of ∠ OPQ.

Solution:	
TP = TQ (Tangents drawn from an exterior point to a circle are equal) Since angles opposite to equal sides of a triangle are equal	
∴ ∠TPQ = ∠TQP	½
In Δ TPQ, 50° + 2 ∠ TPQ = 180°	
⇒ ∠ TPQ = 65°	1
∠ OPQ = 90° − 65° = 25°	½

25. (a) If $\sin 3A = 1$, then find the value of $\cos 2A - \tan^2 45°$.

OR

(b) If $(\sec A + \tan A)(1 - \sin A) = k \cos A$, then find the value of k.

| **Solution:** (a)	$3A = 90° \Rightarrow A = 30°$	½
	$\cos 2A - \tan^2 45° = \cos 60° - \tan^2 45°$	
	$= \dfrac{1}{2} - 1 = \dfrac{-1}{2}$	1½
OR		
(b)	$\left(\dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}\right)(1 - \sin A) = k \cos A$	½
	$1 - \sin^2 A = k \cos^2 A$	½
	$\cos^2 A = k \cos^2 A$	½
	$k = 1$	½

SECTION C

*This section has **6** Short Answer (SA) type questions carrying **3** marks each.*   *6×3=18*

26. A box contains 6 blue, 4 white and 8 red marbles. A marble is drawn at random from this box. Find the probability that the marble so drawn is :
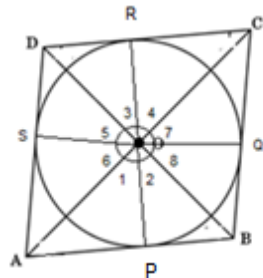
   (i)   white

   (ii)   white or red

   (iii)   not red

Solution: (i)	P (white marble) = $\dfrac{4}{18}$ or $\dfrac{2}{9}$	1
(ii)	P (white or red marble) = $\dfrac{12}{18}$ or $\dfrac{2}{3}$	1
(iii)	P (not a red marble) = $\dfrac{10}{18}$ or $\dfrac{5}{9}$	1

27. In the given figure, a circle is inscribed in a quadrilateral ABCD which touches the sides AB, BC, CD and DA at P, Q, R and S respectively. Prove that $\angle AOB + \angle COD = 180°$.
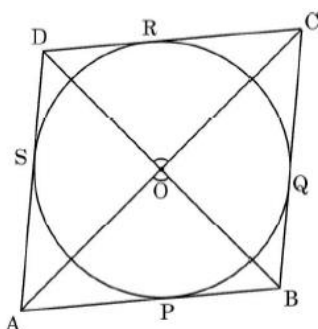
Solution:

Proof of $\triangle OAP \cong \triangle OAS$ (by any congruency criterion) $\Rightarrow \angle 1 = \angle 6$ (cpct)	1
Similarly $\angle 3 = \angle 5$, $\angle 4 = \angle 7$ and $\angle 2 = \angle 8$	1
Also $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$	½
$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$	
$\angle AOB + \angle COD = 180°$	½

28. **(a)** Prove that $\sqrt{2}$ is an irrational number.

OR

(b) Find which among the following numbers a, b and c is/are composite numbers.

$$a = 7 \times 11 \times 13 + 13$$
$$b = 6 \times 5 \times 4 + 4$$
$$c = 7 \times 13 + 6$$

Solution: (a) Let $\sqrt{2}$ be a rational number such that $\sqrt{2} = \dfrac{p}{q}$	½
(p and q are co-prime numbers, $q \neq 0$)	
$\sqrt{2}\,q = p \Rightarrow 2q^2 = p^2$	
2 divides $p^2 \Rightarrow 2$ divides p as well	1
p = 2m (for some integer m)	
$2q^2 = 4m^2 \quad \Rightarrow q^2 = 2m^2$	
2 divides $q^2 \Rightarrow$ 2 divides q as well	
p and q have a common factor 2 which is a contradiction as p and q are co-prime.	1
$\therefore$ our assumption is wrong	
Hence, $\sqrt{2}$ is an irrational number	½
OR	
(b) a and b are **only** composite numbers	3

29. Find the zeroes of the polynomial $25a^2 - 10a + 1$ and verify the relationship between the zeroes and coefficients of the given polynomial.

Solution: $25a^2 - 10a + 1 = (5a - 1)(5a - 1)$	
Zeroes are $\dfrac{1}{5}$ and $\dfrac{1}{5}$	1
Sum of zeroes $= \dfrac{1}{5} + \dfrac{1}{5} = \dfrac{2}{5} = \dfrac{-(-10)}{25} = \dfrac{-Coefficient\ of\ a}{Coefficient\ of\ a^2}$	1
Product of zeroes $= \dfrac{1}{5} \times \dfrac{1}{5} = \dfrac{1}{25} = \dfrac{Constant\ term}{Coefficient\ of\ a^2}$	1

30. (a) A fraction becomes $\frac{1}{3}$, when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$, when 8 is added to its denominator. Find the fraction.

OR

(b) Find the value of k for which the following pair of linear equations will have infinitely many solutions :

$$kx + 3y - (k - 3) = 0 \quad \text{and} \quad 12x + ky - k = 0$$

Hence, find any two solutions of the given pair of equations.

Solution: (a) Let the fraction be $\frac{x}{y}$	½
$\frac{x-1}{y} = \frac{1}{3} \quad \Rightarrow 3x - y = 3 \ldots\ldots\ldots\ldots\text{(i)}$	½
$\frac{x}{y+8} = \frac{1}{4} \quad \Rightarrow 4x - y = 8 \ldots\ldots\ldots\ldots\text{(ii)}$	½
On solving the equations (i) and (ii), we get $x = 5, y = 12$	1
Required fraction is $\frac{5}{12}$	½
OR	
(b) For infinitely many solutions: $\frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$	1
$k^2 = 36$ and $k^2 - 6k = 0$	
$(k = \pm 6)$ and $(k = 6$ or $0)$	
$\therefore k = 6$	1
For k = 6, equations are $6x + 3y = 3$ and $12x + 6y = 6$	
any two correct solutions	½+½

31. Prove the following trigonometric identity :

$$\frac{\tan\theta}{1+\cot\theta} + \frac{\cot\theta}{1+\tan\theta} = \tan\theta + \cot\theta - 1$$

Solution: $\text{LHS} = \dfrac{\tan\theta}{1+\dfrac{1}{\tan\theta}} + \dfrac{\dfrac{1}{\tan\theta}}{1+\tan\theta}$	½
$= \dfrac{\tan^2\theta}{1+\tan\theta} + \dfrac{1}{\tan\theta(1+\tan\theta)}$	½

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$$= \frac{1 + \tan^3 \theta}{\tan \theta (1 + \tan \theta)}$$

½

$$= \frac{(1 + \tan \theta)(1 + \tan^2 \theta - \tan \theta)}{\tan \theta (1 + \tan \theta)}$$

1

$$= \cot \theta + \tan \theta - 1 = \text{RHS}$$

½

SECTION D

This section has 4 Long Answer (LA) type questions carrying 5 marks each.   4×5=20

32.   The following table shows the daily expenditure of 25 households of a locality.

Daily Expenditure (in ₹)	Number of Households
500 – 750	4
750 – 1000	2x + 1
1000 – 1250	12
1250 – 1500	x
1500 – 1750	2

Find the value of x. Hence find the mean daily expenditure.

Solution: $4 + 2x + 1 + 12 + x + 2 = 25 \Rightarrow x = 2$

1

C. I.	x_i	f_i	u_i	$f_i u_i$
500 – 750	625	4	– 2	– 8
750 – 1000	875	5	– 1	– 5
1000 – 1250	**1125** = a	12	0	0
1250 – 1500	1375	2	1	2
1500 – 1750	1625	2	2	4
		25		– 7

Correct Table: **2** marks

$$\overline{x} = 1125 + \frac{-7}{25} \times 250$$

1½

$$\overline{x} = 1055$$

½

∴ mean daily expenditure = ₹1055

33. (a) Find two consecutive odd integers, sum of whose squares is 290.	
OR	
(b) A charity trust decides to build a rectangular hall having an area of 300 m². The length of the hall is one metre more than twice its width. Find the length and breadth of the hall.	

|	
Solution: (a) Let the two consecutive odd integers be x and x + 2	½
$x^2 + (x+2)^2 = 290$	1½
$2x^2 + 4x - 286 = 0$ or $x^2 + 2x - 143 = 0$	1½
$(x - 11)(x + 13) = 0$	
x = 11	1
Required odd integers are 11 and 13	½
OR	
(b) Let width be x m and length be (2x + 1) m	½
A.T.Q. (2x + 1) x = 300	1½
$2x^2 + x - 300 = 0$	1½
$(x - 12)(2x + 25) = 0$	
x = 12	1
(Rejecting $x = \dfrac{-25}{2}$)	
length = 25 m and width = 12 m	½

34. (a) State and Prove "Basic Proportionality Theorem".
OR
(b) In the given figure, CM and RN are respectively, the medians of △ ABC and △ PQR. If △ ABC ~ △ PQR, prove that :
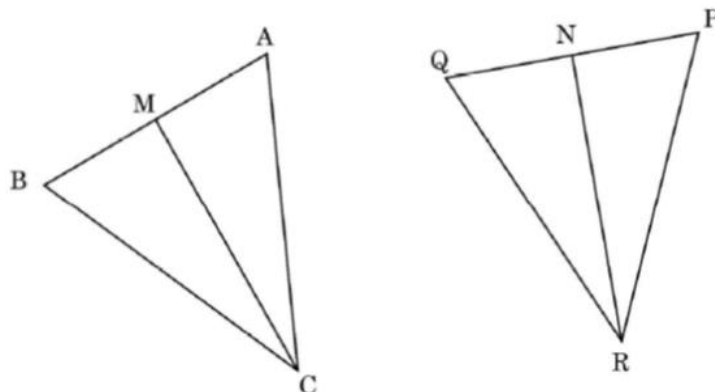
Solution: (a) Statement: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.	Correct Statement: 1 mark

Given: In $\triangle$ ABC, DE $\parallel$ BC

To Prove: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Given + To prove + Construction + Figure: 1 mark
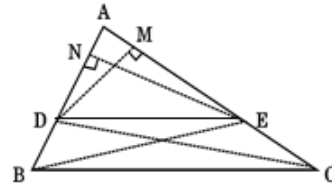
Construction: Draw DM $\perp$ AC, EN $\perp$ AB, join BE and CD

Proof : $\dfrac{ar(\triangle ADE)}{ar(\triangle DBE)} = \dfrac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \dfrac{AD}{DB}$ ………………..…(i)   **1**

$\dfrac{ar(\triangle ADE)}{ar(\triangle ECD)} = \dfrac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \dfrac{AE}{EC}$ …………..………..(ii)   **1**

as $\triangle$DBE and $\triangle$DCE lie on the same base DE and between same parallels BC and DE

$\therefore$ ar($\triangle$DBE) = ar($\triangle$ECD) or $\dfrac{ar(\triangle ADE)}{ar(\triangle DBE)} = \dfrac{ar(\triangle ADE)}{ar(\triangle ECD)}$ ……….(iii)   **½**

From (i), (ii) and (iii), we get $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ **½**

OR

(b)   (i) $\triangle$ABC ~ $\triangle$PQR $\Rightarrow \dfrac{AB}{PQ} = \dfrac{AC}{PR}$ **½**

$\Rightarrow \dfrac{AC}{PR} = \dfrac{\frac{1}{2} AB}{\frac{1}{2} PQ} \Rightarrow \dfrac{AC}{PR} = \dfrac{AM}{PN}$ **½**

Also $\angle$ A = $\angle$ P   **½**

$\therefore \triangle$AMC ~ $\triangle$PNR (by SAS similarity criterion)   **½**

(ii) $\triangle$AMC ~ $\triangle$PNR (from part (i))

$\therefore$ $\angle$ACM = $\angle$PRN   **½**

Also $\angle$ACB = $\angle$PRQ (as $\triangle$ABC ~ $\triangle$PQR)

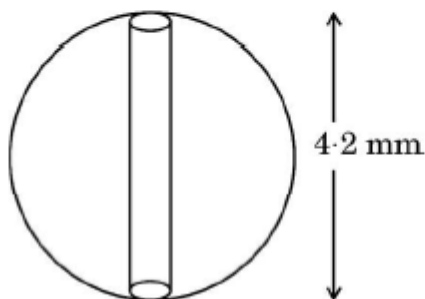|	
$\therefore \angle ACB - \angle ACM = \angle PRQ - \angle PRN$	½
$\Rightarrow \angle BCM = \angle QRN$	
(iii) $\triangle ABC \sim \triangle PQR \Rightarrow \dfrac{AB}{PQ} = \dfrac{BC}{QR}$	½
$\Rightarrow \dfrac{BC}{QR} = \dfrac{\frac{1}{2}AB}{\frac{1}{2}PQ} \Rightarrow \dfrac{BC}{QR} = \dfrac{BM}{QN}$	½
Also $\angle B = \angle Q$	½
$\therefore \triangle BMC \sim \triangle QNR$ (by SAS similarity criterion)	½

35. A necklace is made up of wooden beads. Each bead is in the form of a sphere of diameter 4·2 mm. A cylinder is hollowed out from each bead. If the radius of the cylinder is 1 mm, find the volume of wood left in each bead.

4·2 mm

Solution:

Radius of a spherical bead = R = 2·1 mm

radius of cylinder = r = 1 mm

height of cylinder = h = 4·2 mm

Volume of wood left in a bead

= Volume of sphere − Volume of cylinder

$= \dfrac{4}{3}\pi R^3 - \pi r^2 h$

$= \dfrac{4}{3} \times \dfrac{22}{7} \times 2.1 \times 2.1 \times 2.1 - \dfrac{22}{7} \times 1 \times 1 \times 4.2$ 2 + 2

= 25.608 cu. mm   1
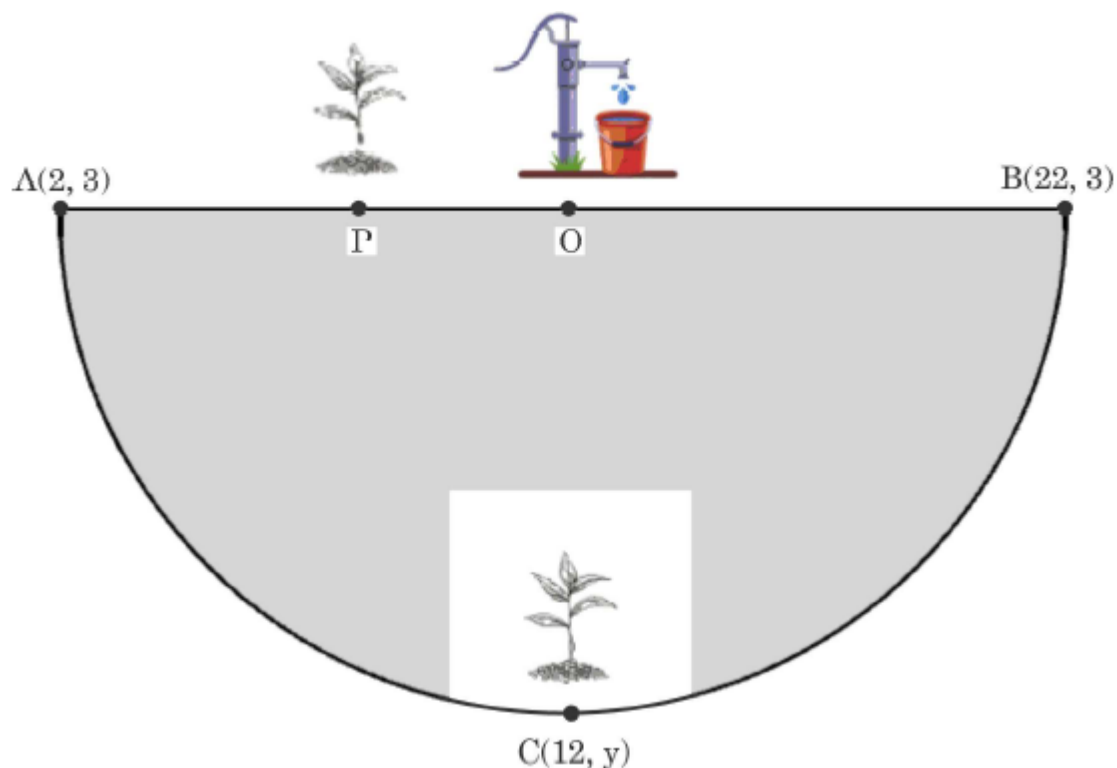
SECTION E

This section has 3 case study based questions carrying 4 marks each.   3×4=12

Case Study – 1

36. There is a semicircular park in Aman's society. He wishes to plant saplings along the boundary of the park. There is a borewell at the centre O of the park along the diameter AB as shown in the figure below.

A(2, 3)   P   O   B(22, 3)

C(12, y)

Based on the above information, answer the following questions :

(i) Find the coordinates of point O. *1*

(ii) Find the radius of the semicircular park. *1*

(iii) (a) One sapling is kept at point C(12, y). Find the coordinates of C. *2*

OR

(b) One sapling is kept at point P along AB so that PA = $\frac{1}{3}$ PB. Find the coordinates of P. *2*

Solution:

(i) Coordinates of O are (12, 3) — 1

(ii) Radius = 10 — 1

(iii) (a) OC = radius = 10 — ½

y = 13, y = – 7 — 1

Coordinates of the point 'C' are (12, 13) or (12, –7) — ½

OR

(iii) (b) P divides AB in the ratio 1 : 3 — ½

Coordinates of P are $\left(\dfrac{1 \times 22 + 3 \times 2}{4}, \dfrac{1 \times 3 + 3 \times 3}{4} \right)$ — 1

i.e. (7, 3) — ½

Case Study – 2

37. In a society, a yoga instructor was hired to train the people of the society to live a healthy lifestyle. Yoga sessions were held daily from 5 p.m. to 7 p.m. in the society park. On day one, 5 people joined the yoga session, on day two, 3 more people joined, on day three, another 3 people joined and in this manner every next day, 3 more people kept on joining.

Based on the given information, answer the following questions :

(i) On which day did 59 people join the yoga session ?   *1*

(ii) How many people joined the yoga session on the 31st day ?   *1*

(iii) (a) The yoga instructor was paid ₹ 100 for each person attending the yoga session. On which day would he earn ₹ 5,000 ?   *2*

OR

(b) What was the total amount earned by the yoga instructor in 16 days ?   *2*

Solution:

(i) $5 + (n - 1)\, 3 = 59$

$\quad n = 19$ 1

(ii) $a_{31} = 95$ 1

(iii) (a) Number of persons $= \dfrac{5000}{100} = 50$ 1

$\quad 5 + (n - 1)\, 3 = 50$

$\quad\quad n = 16$ 1

	OR	
(iii)	(b) $S_{16} = \dfrac{16}{2}[\,10 + 15\,(3)]$	1
	$= 440$	½
	Total amount earned in 16 days $= 440 \times 100$	
	$= ₹\ 44000$	½

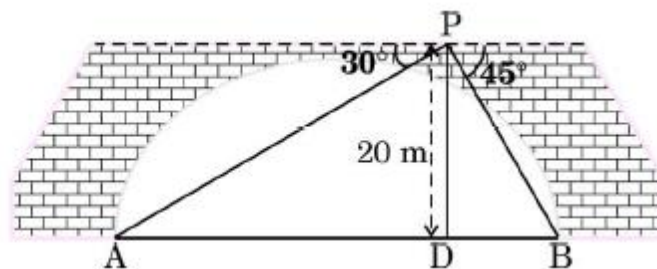<div align="center">Case Study – 3</div>

38. Two motorboats A and B are waiting at the opposite banks of a river in order to reach the opposite side. From a point P on the bridge, 20 m above the river, the angles of depression of the boats are 30° and 45° respectively, as shown in the figure given below. Both the boats leave at the same time at the speed of 10 m/s and 5 m/s, respectively

Based on the above information, answer the following questions :

(i)	Find the distance travelled by boat A to reach point D in the river, vertically below the point P. (Use $\sqrt{3} = 1\cdot73$)	*1*
(ii)	What is the width of the river ?	*1*
(iii) (a)	Which boat will reach point D first, and how much earlier, than the other boat ?	*2*
	OR	
(b)	What is the distance between the two boats after 3 seconds ?	*2*

Solution:		
(i)	$\dfrac{20}{AD} = \tan 30°$	½
	$AD = 20\sqrt{3} = 34.6$ m	½
(ii)	$\dfrac{20}{DB} = \tan 45° \Rightarrow DB = 20$ m	½
	Width of river $= 34.6 + 20 = 54\cdot6$ m	½
(iii) (a)	Time taken by boat A $= \dfrac{34\cdot6}{10} = 3.46$ seconds	1

Time taken by boat B = $\dfrac{20}{5}$ = 4 seconds		½
Boat A will reach earlier by 0·54 seconds		½
OR		
(iii) (b) Distance covered by boat A in 3 seconds = 3 × 10 = 30 m		½
Distance covered by boat B in 3 seconds = 3 × 5 = 15 m		½
Distance between them after 3 seconds = 54.6 – (30+15)		
= 9·6 m		1